Directions: If units are appropriate include them in your answer.

1. Solve for $x$: \[ 1 + \frac{1}{1 + \frac{1}{x}} = \frac{15}{8}. \]

2. Bruce travelled 10,000 miles in a car with one spare tire. He rotated the five tires at intervals so that when the trip ended each tire had been used for the same number of miles. For how many miles was each tire used?

3. Consider the system of equations
   \[
   \begin{align*}
   \frac{1}{a} + \frac{1}{b} &= \frac{2}{7} \\
   a + b &= 32,
   \end{align*}
   \]
   where $a$ and $b$ are positive integers and $a < b$. Find $b$.

4. Hasse is twice as old as Duke. Four years ago, Duke was twice as old as Maisy. In 10 years Hasse will be twice as old as Maisy. How many years old is Hasse now?

5. The letters A, B, and C represent three different digits. Determine the number ABC if ABC + ACB = BCA.

6. If $N$ is a positive integer, then $N!$ (read $N$ factorial) is the product of the integers from 1 to $N$. For example $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$. Find the smallest $k$ such that the product $N!$ never ends in $k$ zeroes for any $N$.

7. A pair of positive integers has a product of 1,000,000 and yet neither integer has a zero as one of its digits. What is the smaller of these two positive integers?

8. Find all possible values of $c$ such that the system of equations
   \[
   \begin{align*}
   y &= -x + 6 \\
   y &= \frac{1}{3}x + c
   \end{align*}
   \]
   has its solution in quadrant I (i.e. $x \geq 0$ and $y \geq 0$).
9. A jar contains some red and some yellow jelly beans. If Daisy were to eat one red jelly bean, then \( \frac{1}{7} \) th of the remaining jelly beans would be red. If instead Daisy were to eat 5 yellow jelly beans then \( \frac{1}{6} \) th of the remaining jelly beans would be red. How many jelly beans are in the jar?

10. Find the minimum possible value of \( 2x^2 + 2xy + 4y + 5y^2 - x \) for all real numbers \( x \) and \( y \).

11. The quadratic \( x^2 - 4x - 5 \) is a factor of the cubic \( ax^3 + bx^2 + 25 \). Compute the value of \( a - b \).

12. Find the number of different ordered pairs \((a, b)\) of integers such that \( ab + a - 3b = 5 \).

13. Consider the linear equation \( 5x + 11y = 2011 \). How many solutions \((x, y)\) are there such that \( x \) and \( y \) are positive integers?

14. Find the positive integer \( n \) such that \( n^3 + 4n^2 + 4n - 1859 = 0 \).

15. Find the largest integer that divides \( n^5 - 5n^3 + 4n \) for every integer \( n \).

16. Hasse has 200 coins. He wishes to distribute all the coins to friends in such a way that each friend gets at least one coin and no two friends get the same number of coins. What is the largest number of friends he can use?

17. Find all real numbers \( x \) such that \( \sqrt{1 + \sqrt{x}} = x - 1 \).

18. Let \( n = 1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot 2009 \cdot 2011 \). Note that \( n \) is the product of the odd integers from 1 to 2011. Find the last (units) digit of \( n \).

19. Find all integers \( x \) in the interval \([20, 40]\) such that 10 divides \( 6x + 24 \).

20. Find the value of \( x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}} } \).

21. King Midas spent \( \frac{100}{x} \) \% of all his gold yesterday. (He did not spend all of his gold.) What percentage of the amount of gold King Midas currently has would he have to obtain today in order for him to have as much gold as he had at the beginning of yesterday?

22. Daisy and Duke each randomly choose a whole number between 1 and 1000 inclusive. What is the probability that Duke’s number is larger than Daisy’s?